Finite Automata – Introduction

Lecture 4
Section 2.1

Robb T. Koether

Hampden-Sydney College

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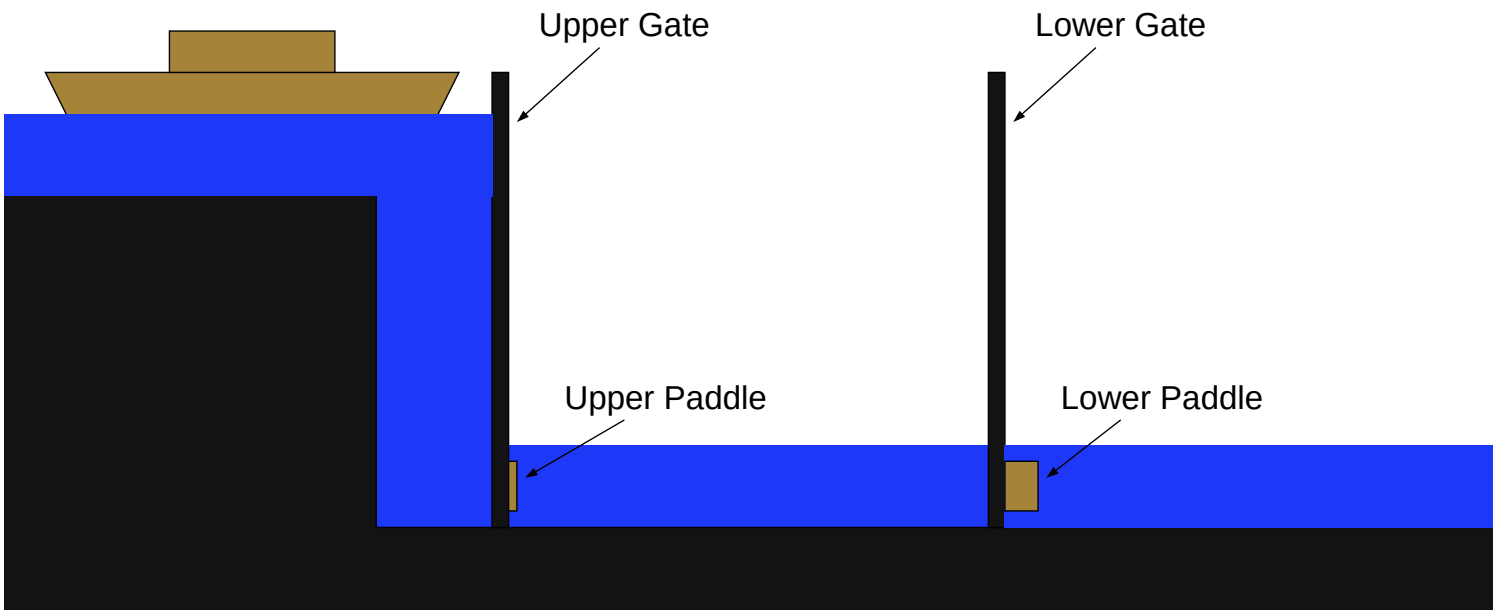
Outline

Outline

A Canal Lock

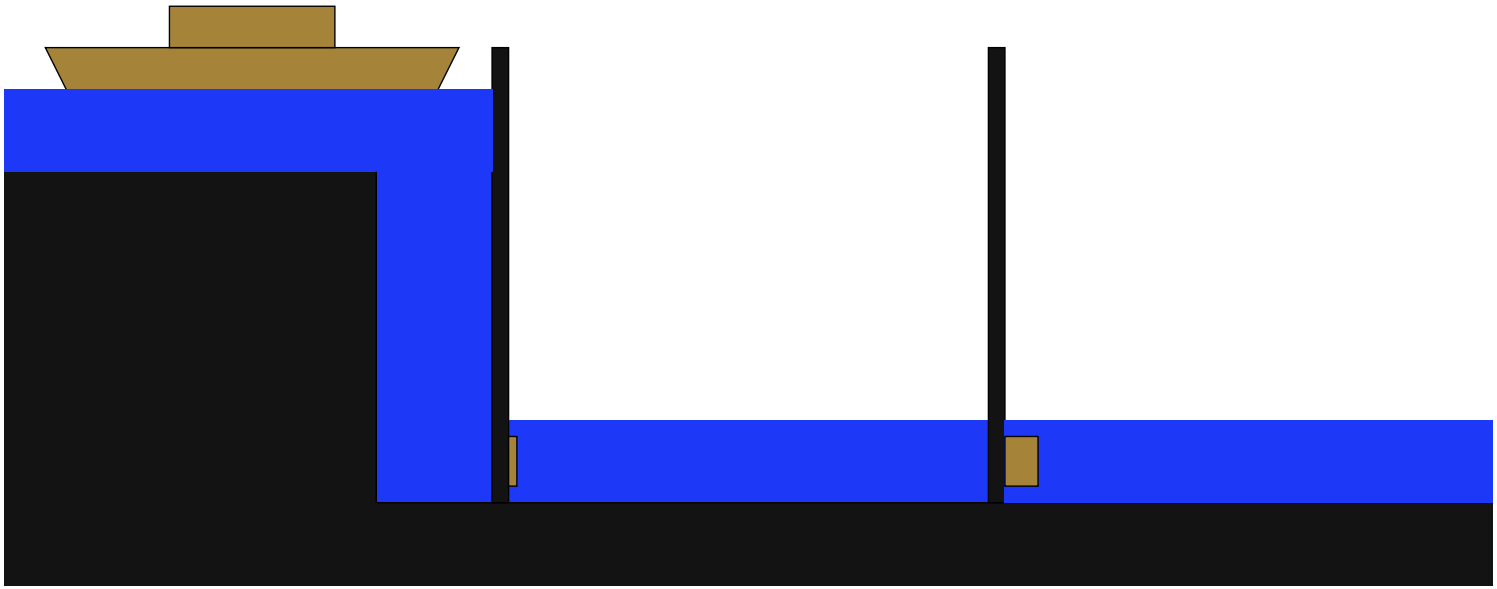
- Describe the operation of a canal lock operated so that the gates can never be opened when the water on the two sides of the gate is not at the same level.



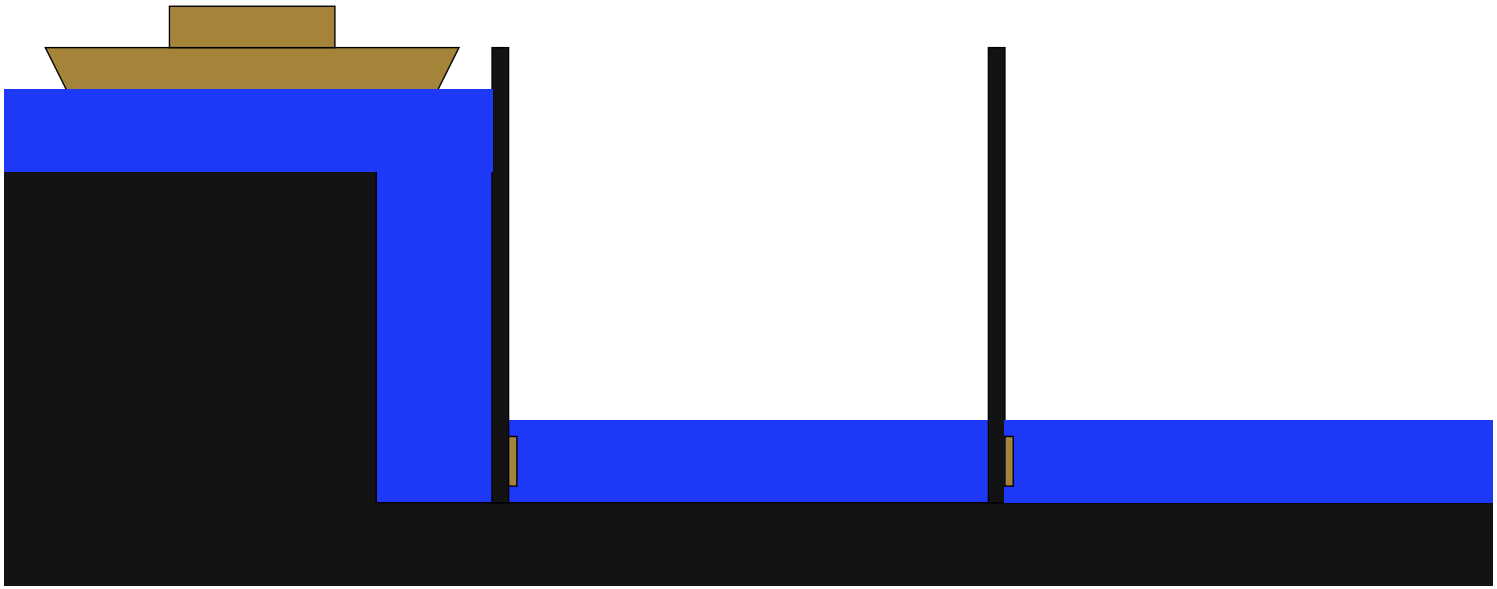
A Canal Lock

- The working parts of the lock are
  - Upper gate
  - Upper paddle
  - Lower gate
  - Lower paddle
- Each gate or paddle is either open or closed.

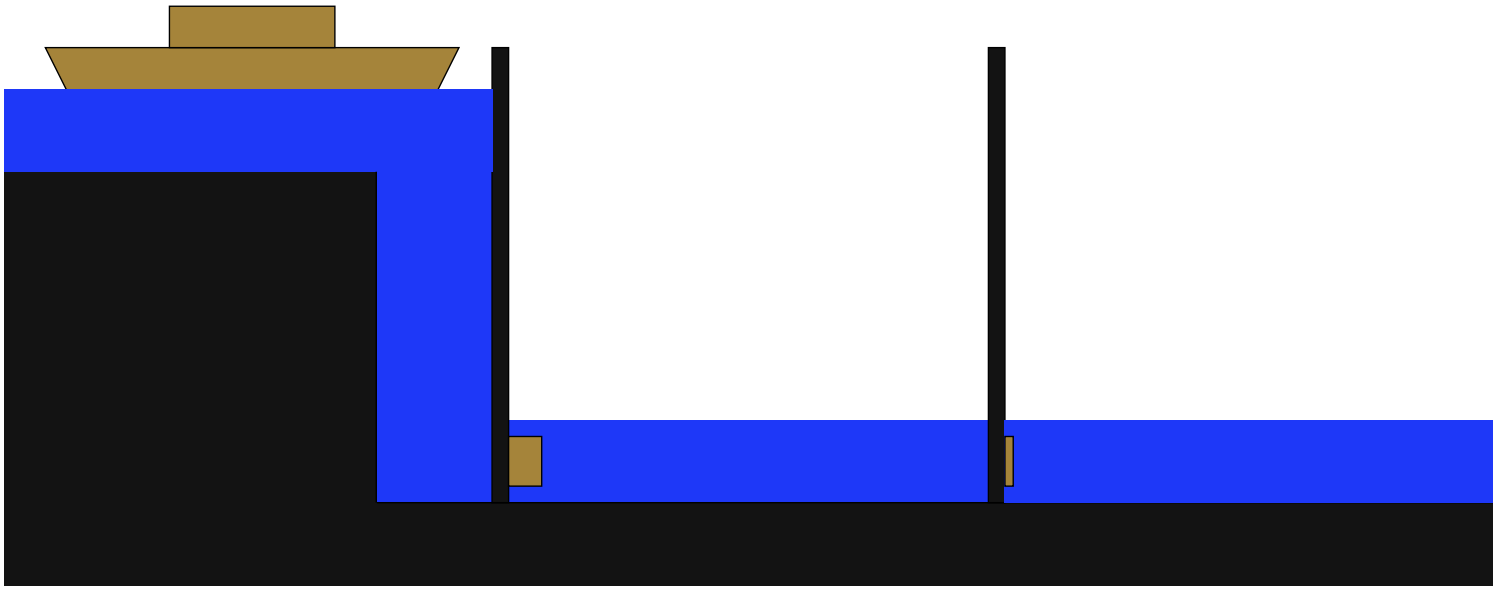
A Canal Lock



A Canal Lock



A Canal Lock



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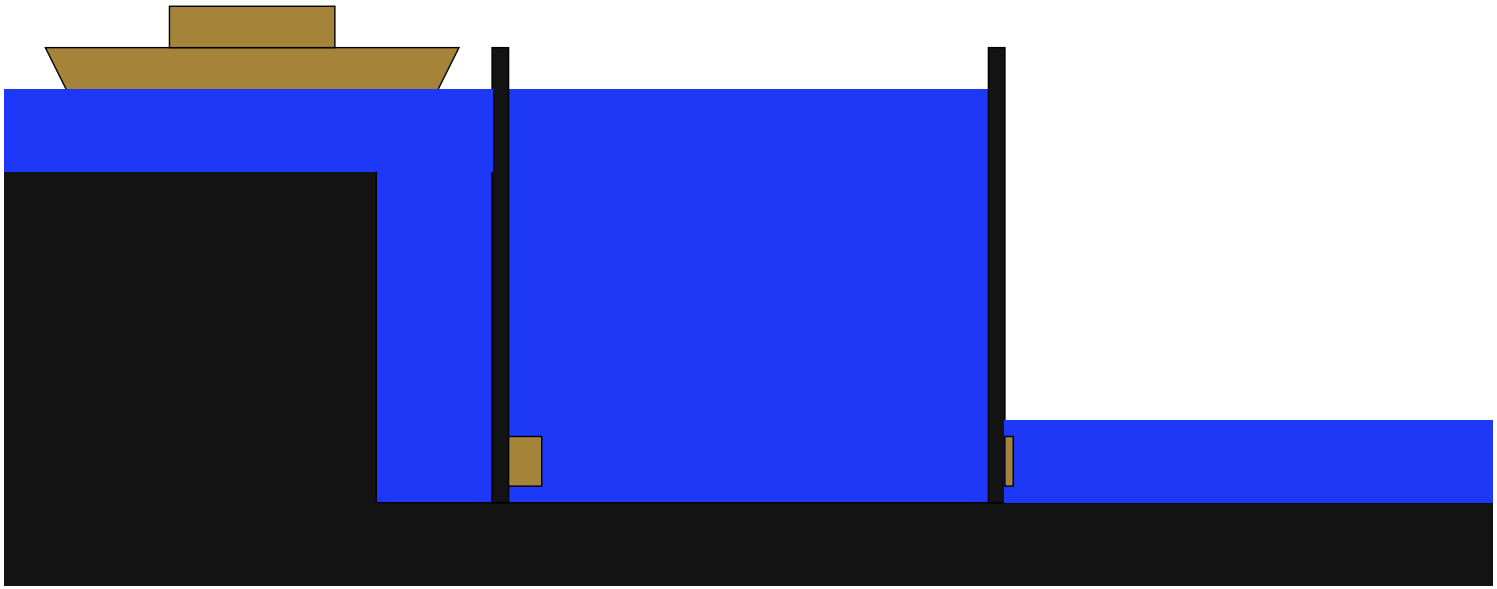
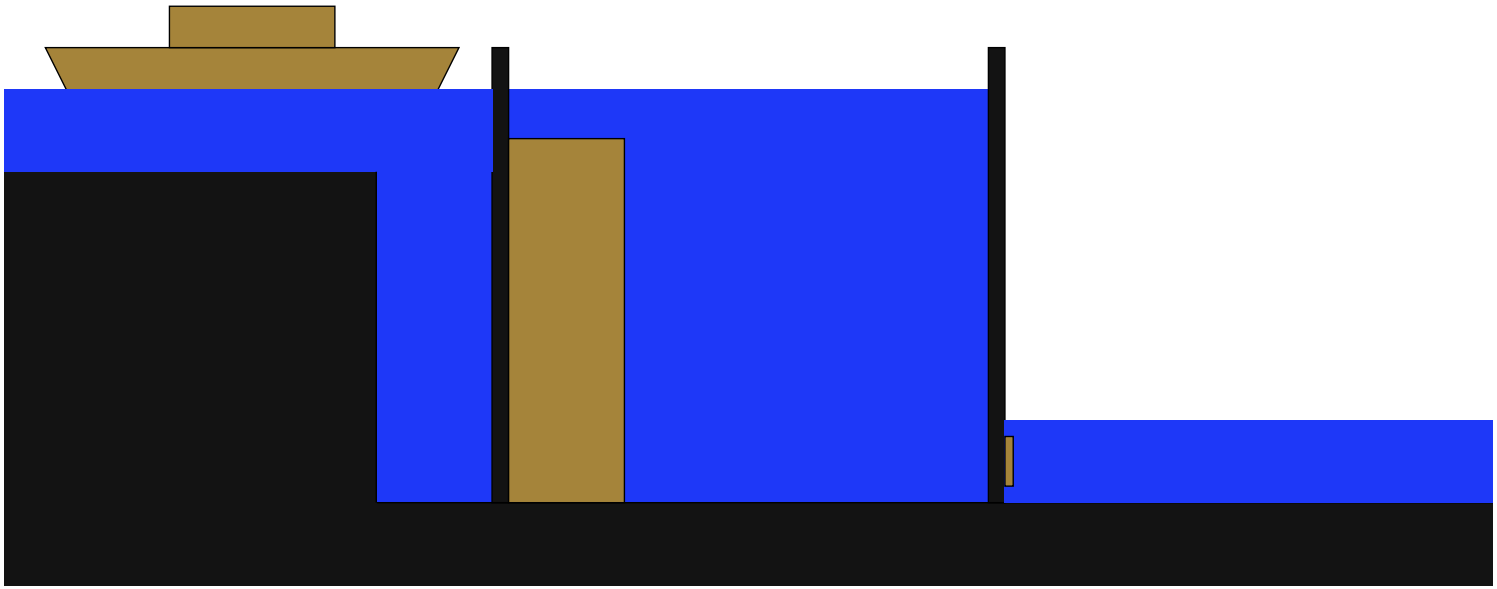
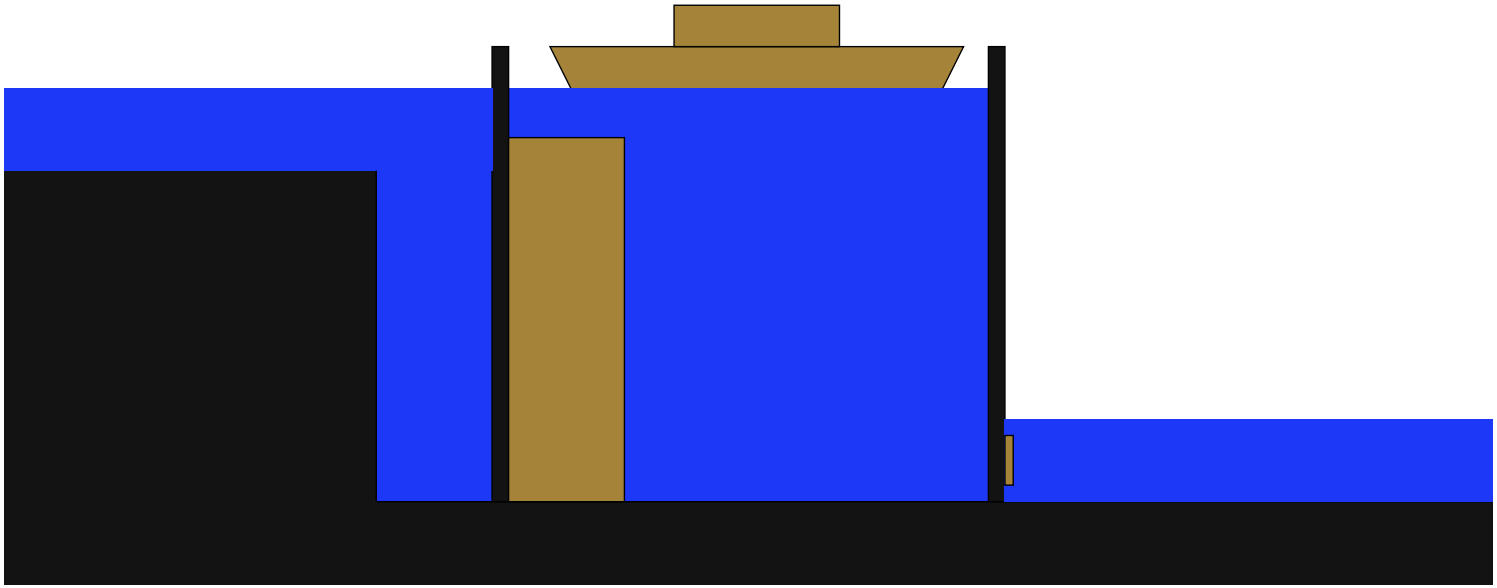


A Canal Lock



A Canal Lock



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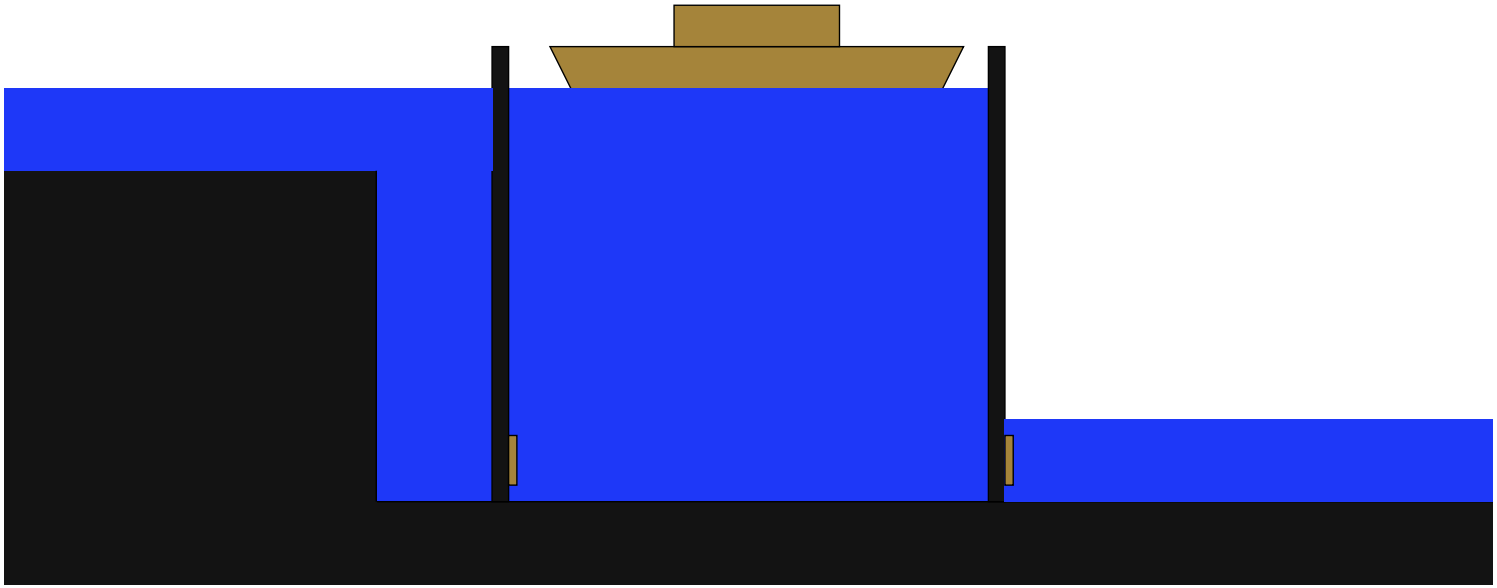

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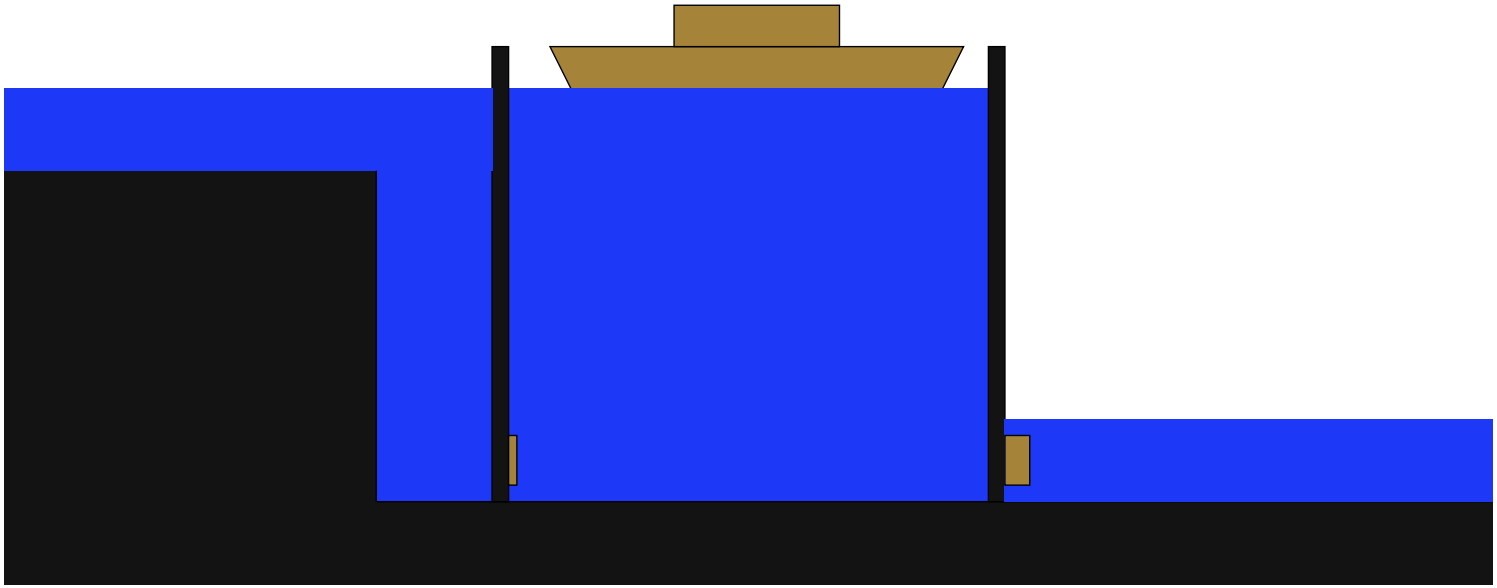

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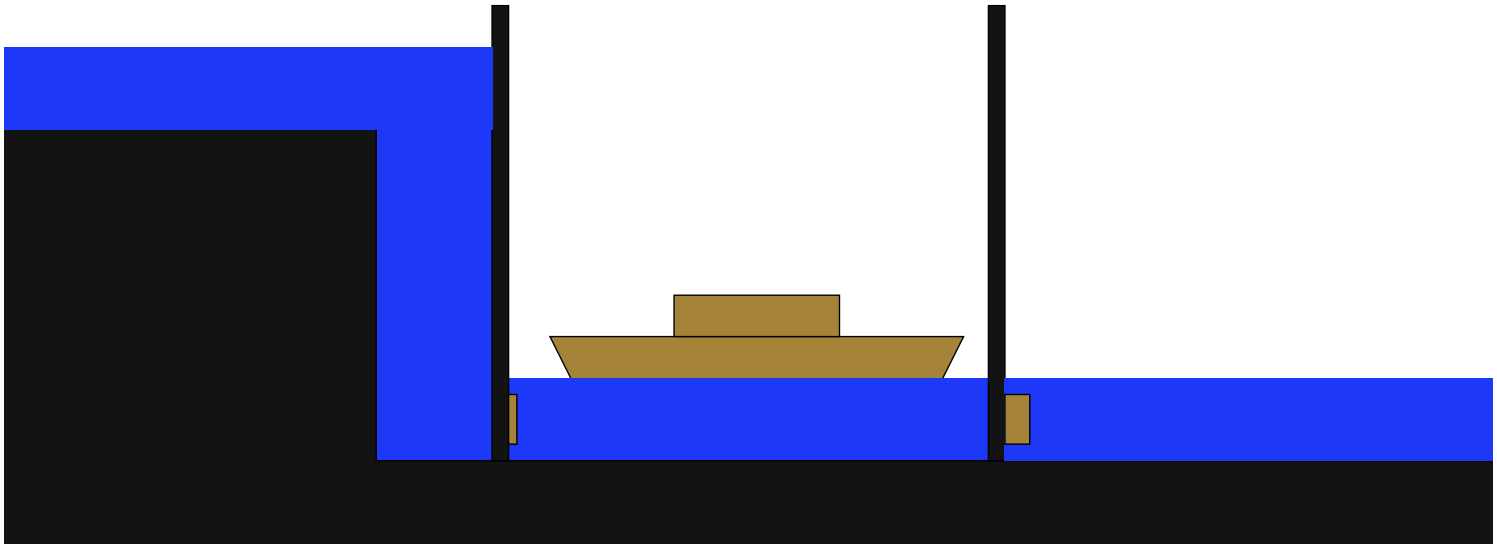

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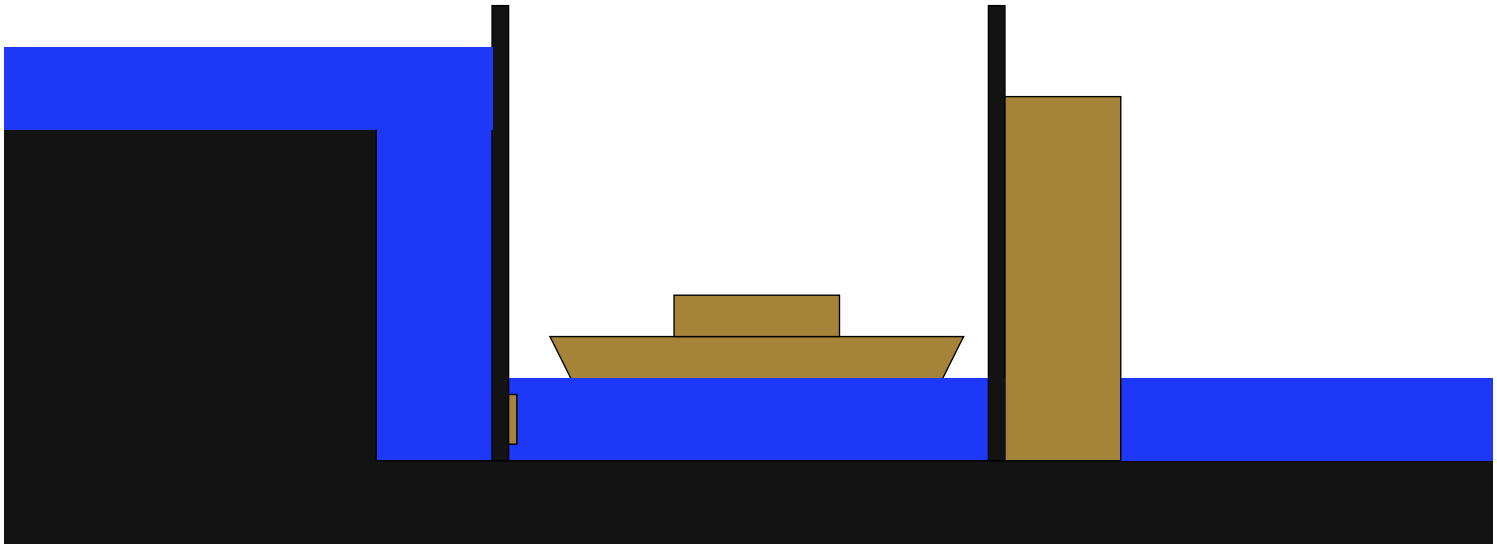

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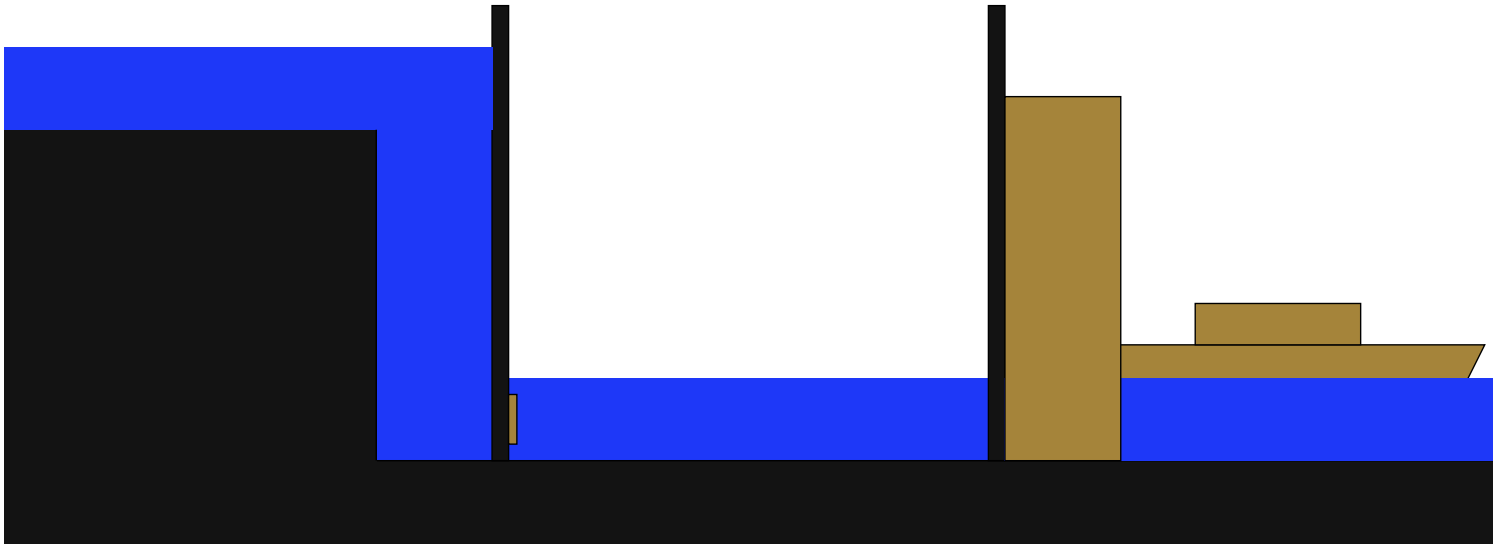
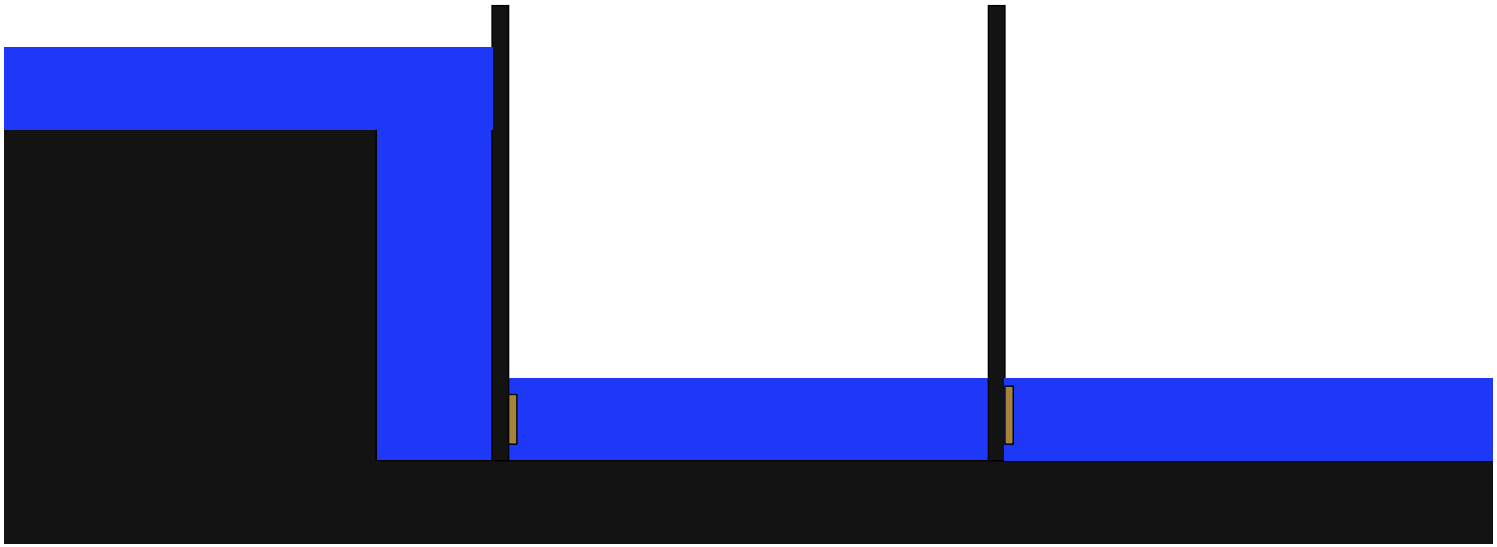


A Canal Lock



A Canal Lock

- What are the states?
- Each gate or paddle is open or closed.
- So there are 16 conceivable combinations.
- How many are feasible? (We can never have an upper gate or paddle open while a lower gate or paddle is open.)
- Also, the water can he high or low, but that is typically determined by the state of the gates and paddles.

A Canal Lock

- The following table shows the possible states (X = closed, O = open).

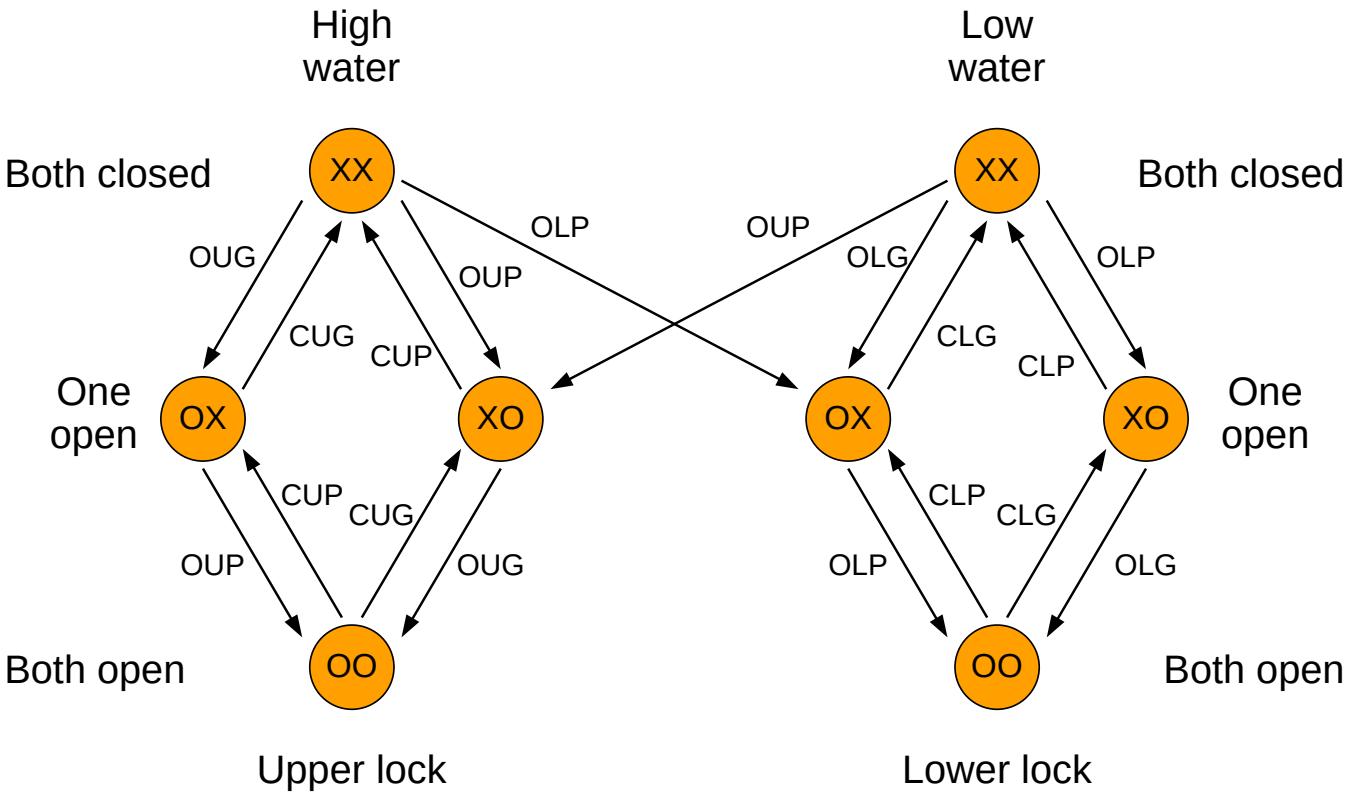
State	UG	UP	LG	LP	Water
1	X	X	X	X	Low
2	X	X	X	X	High
3	X	X	X	O	Low
4	X	X	O	X	Low
5	X	X	O	O	Low
6	X	O	X	X	High
7	O	X	X	X	High
8	O	O	X	X	High

A Canal Lock

- Draw a state diagram where the actions are
  (a) Open upper gate.
  (b) Close upper gate.
  (c) Open upper paddle.
  (d) Close upper paddle.
  (e) Open lower gate.
  (f) Close lower gate.
  (g) Open lower paddle.
  (h) Close lower paddle.
- Show only feasible transitions.

A Canal Lock

- The state diagram for the canal lock.



A Canal Lock

- Describe how to move a boat downstream through the locks where the initial conditions are
  - All gates and paddles are closed.
  - The water level is low.

  and the final conditions are the same.

A Canal Lock

- Describe how to move a boat upstream through the locks where the initial conditions are
  - All gates and paddles are closed.
  - The water level is high.

  and the final conditions are the same.

Outline

Definition of a Finite Automaton

Definition (Deterministic finite automaton)

A deterministic finite automaton, written DFA, is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$, where

- Q is a finite set of states,
- Σ is a finite alphabet,
- $\delta : Q \times \Sigma \to Q$ is the transition function,
- $q_0 \in Q$ is the start state, and
- $F \subseteq Q$ is the set of accept states.

Outline

Examples

Example (Definition of a DFA)

- Describe the canal locks formally.
  - Let the start and end states be that the gates and paddles are closed and the water is low.

Definition (Language of a DFA)

The language of a DFA M is the set of all strings that are accepted by that language.

$$L(M) = \{w \in \Sigma^* \mid M \text{ accepts } w\}.$$

Definition (Regular language)

A language L is regular if there exists a DFA M such that $L = L(M)$.

Examples

Example (Examples)

Design DFAs that will recognize the following regular languages over $\Sigma = \{\mathbf{a}, \mathbf{b}\}$.

- All strings that start with **a**.
- All strings that end with **a**.
- All strings that contain **aaa**.
- All strings in which each **a** is followed immediately by **b**.
- All strings that contain **aba** or **bab**.
- All strings that contain **aba** and **bab**.

Examples

Example (Binary Addition)

- Design a DFA that will recognize mathematically correct binary addition problems.
- For example:

$$\begin{array}{r} 10110 \\ \underline{00100} \\ 11010 \end{array}$$

- The input symbols are triples of binary digits (000, 001, 010, etc.), representing the columns.
- Read the columns from right to left.

Examples

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- For example:

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- The input symbols are triples of binary digits (000, 001, 010, etc.), representing the columns.
- Read the columns from right to left.
- Can we also process them from left to right with a DFA?

Outline

Assignment

Assignment

- Section 2.1 Exercises 1, 2, 3, 4bc, 7ef, 8a.